Reasoning system to derive spatial information out of mixed quantitative-qualitative representations

IRTG Thesis Proposal

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1 Introduction

Spatial data representation and the possibility of making spatial reasoning is essential in the field of Artificial Intelligence.

There has been considerable research on spatial representations which are based on metric measurements, this research has led to the development of Geographic Information Systems (GIS). In the broadest sense, the term describes any system that integrates information, stores, edits, analyses, shares, and displays geographic information [1].

A different way to express spatial information makes use of qualitative representation of data. In these cases the exact location or the exact shape of an object are not significant; what is more interesting are qualities and relations that this object realizes with other regions in space. This kind of representation can be seen as an abstraction out of quantitative representation of spatial data. Dealing with qualitative representation is motivated by a wide range of applications, like robot navigation tasks, natural language interpretation, dealing with partial descriptions of the environment.

For some purposes it is enough to only represent spatial knowledge, but what makes intelligent systems intelligent is their ability to also reason about the given knowledge [6].

Typically it is easy to find techniques to move from a geometric representation of spatial information to a qualitative model that describes some characteristics of interest. Indeed, in many cases the identification of mathematical rules is enough to verify some relationship between regions described in a quantitative way (e.g. by drawing tangents, measuring distances, comparing the boundary,).

The reverse operation, namely to find a geometry starting from a qualitative one, is more complex because qualitative descriptions provide "partial" information regarding the real location and shape of the object.

The proposed work lies in an intermediate position between the two different techniques of representing spatial data. The representation in this context can be defined as mixed, some regions of space are represented in quantitative terms, while others are represented in qualitative terms.

The target of the proposed work is to develop a reasoning system to derive new information from the given one. Derived information is both qualitative and quantitative. This new information, together with the given one, will be useful to solve tasks related to visualization (e.g. maps, graphics, etc.) or querying (e.g. support for decision making problems). It is assumed that given qualitative relations can refer both to objects geometrically defined and to objects described in qualitative terms.

It is important to consider the semantic of the given information during the integration process made by the reasoning system. Given data can come from heterogeneous sources, like humans or sensors, and can assume different meanings depending on their origin.

In order to describe the advantages of this approach it is possible to consider the following example: suppose that one wants to build a spatial database describing an environment. Some objects already are described geometrically by their location and shape (eg houses, buildings, etc.), while for other objects there is no precise knowledge about shape and position (eg forests), because for any reason it is not possible to measure directly the geometric characteristics of such objects. In these cases, using a traditional approach, it is not possible to describe objects for which geometric characteristics are not know. Instead, using qualitative descriptions of these objects (eg proximity to other objects, direction, ...) and using the described approach, it may be possible to approximate the real extension of the objects using them as any other objects within the geographical database.

As previously specified qualitative descriptions may include relations that describe objects with respect to both other objects geometrically described (simple case) and to items also described in a qualitative way (more complex case).

The following sections will describe the proposed work more in detail. In particular in Section 2 an application scenario, that will be used as basis for the whole work, is introduced. Section 3 contains a brief description of the state of the art, while Section 4 describes what will be the contribution of the proposed work. Finally in Section 5 a practical approach to solve problems described in the Contribution section is proposed.

2 Application scenario - Disaster Management (Earthquake)

In the early morning hours of 6 April 2009 a severe earthquake hit the city of L'Aquila and its surroundings, in the Abruzzo region of Italy, causing the death of 307 people and more than 1'500 injured. Between 3'000 and 11'000 buildings were damaged solely in the city of L'Aquila.

2.1 General scenario

Typically, urban and wild environments are represented in GIS application developed by national or local administrations, industries, military forces, etc. Each of them stores in a GIS system all the geographical informations useful for their particular goals: a local administration could have a good description of roads, buildings, natural zones belonging to the environment; an industry enterprise wants to manage, in most cases, information useful for their kind of business; military forces, indeed, would like to store and manage information related to strategic points or risk zones within an urban environment. Obviously the cited application are only a small part of the global GIS purposes and applications.

With full access to all the datasets stored by the different administrations or industries GIS, it is possible to say that one can solve almost all kind of problems and can develop all kind of geographical applications using the informations stored in existing GIS.

Data changes are often slow processes and changes affect only a minimal part of the whole data stored in a GIS; let us imagine the process of construction of a new road or building that requires a long time to be completed and that affects only the neighbour of the new entity.

In this case it is quite easy to update GIS information: the only mandatory operation is to make accurate measure of the new entity and add it to the system, and eventually update information related to objects that was affected by the changes.

But what happens when an unexpected natural event, like an earthquake or a tornado, changes the typical static environment in a few seconds? This kind of event can cause a lot of changes in the environment: a bridge can fall down, a landslide can occlude a road, etc.

In this case, different from the typical situation described before, data changes are no more slow processes and can affect a big part of the whole data managed by GIS.

Obviously these kind of changes make the environment description stored in a GIS system unusable; at the same time, after a natural catastrophe GIS data is particularly important as it is needed to coordinate the different aid operations. In particular, considering the example of an earthquake hitting a city, it is possible to find a lot of tasks that need to start in the minutes after the natural event and need to be completed in the shortest possible time. Let us consider, for example, crucial operations like victims rescue, aid dispatch to survivors, organization of gather points and of management points for people who have to manage the catastrophic situation.

The requirement of first aid makes an efficient infrastructure necessary. The fire-fighters and rescuers rely on a flow of latest information. The operational command has to update its information to avoid losing time by having emergency vehicles reaching deadlocks.

With a good description of the environment, it is possible to take care of these kind of tasks reducing the risk for rescue missions and for survivors; in contrast, using information stored in a non-updated GIS, e.g. to find the shortest way to reach a zone where there is necessity of aid, there are intrinsic risks to obtain a path that can not be used anymore, because of collapses or landslide; this eventuality increase hazard for rescue missions and also increase the time to take aid to people.

The challenge is then to try to update informations on GIS systems in a fast way, using when possible automatic agents instead of humans and take advantage from the power of modern automatic calculators.

Modern observation systems make a quick surveillance possible. Remote sensing, unmanned aerial vehicles, but also reports by helpers and victims provide valuable information. Different groups, like the military, local administrations, the Red Cross and other aid organizations, universities or single persons can contribute valuable information. An efficient integration of various sensory information sources across communities and different data models is crucial in this situation. The red cross does not only need to know how many people are injured, but they need to decide on where to set up their camps and mobile hospitals to achieve the best accessibility. Blocked roads have to be removed from the route planning. Helicopter landing fields have to be scouted. Medical supply has to be delivered. In such case medical information, demographic information, military information and personal information have to be integrated and made available to the institutions that need them. In time critical applications it is not only important to make all information available to all parties, but to support the retrieval of precisely the required information.

2.2 Hybrid Quantitative and Qualitative Reasoning

The contribution proposed in this paper has as target to help the operation of reconstruction of the changed environment map.

Obviously a natural event, like an earthquake, can change a lot of entities that are part of an urban environment, but there are still some entities that may not have been affected by the catastrophe (e.g. a particular building or a particular historical monument). The main idea is to send some survey agents (humans or automatic) that start to explore the changed environment, and they describe it to a central system in a simple and natural language from their perspective. For example an agent can say that he sees a collapse in front of the main square of the city, or that there are survivors between two collapsed buildings. Of course each agent will use a different way to describe the environment depending on his background and his task. The presence of fire will be probably described in different ways by firemen and red cross operators.

The central system will then process the received informations and try to derive new information to help rescue and aid operations. One useful task is to reconstruct the new map of the environment as good as possible, making approximation when needed and trying to extrapolate new informations from the information given by the agents. This new map will be used for all the rescue operations cited before. Also a support to decision making tasks (e.g. path finding, settlements of rescue camps, etc.) can be furnished. Presentation of data can differ depending on the task the final user has to achieve and of course from his cultural background. Obviously the new information can not be precise, but still reduce risks for rescue missions and reduce time to send aid where it is needed, without scattering rescue forces in useless ways.

Problems involved in the process of description's reconstruction of the changed environment are various and of different nature. Below these problems are briefly described.

First of all the process of recognition of known places has to be considered or, in other words, the process that determine all the entities that did not undergo changes after the natural catastrophe.

The second task that can be identified regards how human and automatic agents communicate with the central system, furnishing information regarding the description of the environment from their point of view. Humans use typically a natural language to communicate and automatic agents use a mathematical language. It has to be noted that different human agents can refer to the same thing using different expression depending on their culture and tasks they are involved in. A fireman will describe a fire in a different way it is described by a red cross operator, due to his experience and knowledge in the field.

Humans expressions give typically qualitative descriptions instead of quantitative ones and also they give only a partial description of the entities. The system that reconstructs the map has to take these aspects into account.

Finally the last task is related to the final reconstruction of map. At this point it is possible to suppose to have a set of known entities (quantitatively described), a set of unknown entities and a set of relations that relate entities belonging to the two previous sets that give a qualitative description of the unknown entities. The target is to infer new information from the given one. One operation that can be addressed is to create a quantitative description of unknown entities as close as possible to the real characteristics of the entity. It is also possible to find out new qualitative relations among known and unknown entities. Problems to take into account concern the time performance requirements (the process of reconstruction has to be fast to help rescue missions) and also problems of consistence (descriptions given by different agents can contradict each other).

Within the proposed work only the last task will be addressed. The suppositions are that the description of known and unknown entities is available and also problems regarding interpretation of natural language and communications among humans and automatic agents are solved. Characteristics arising from this steps are still to be considered in the phase of map reconstruction, like the qualitative and incomplete description of environment; these characteristics will bias how the map reconstruction task will be developed.

3 State of the art

Spatial representation and spatial reasoning are fundamental elements for all those applications, in the artificial intelligence field, that requires a model of some particular spatial aspects of the real world.

Fields of research, such as geographic information science (GIScience) [9], led to produce efficient, expressive and useful spatial calculi. There is considerable research in spatial representation that is based on metric measurements, e.g. in the vision [7] [15] and robotics communities [23], and also on raster and vector representations in GIScience [25].

Another way to represent spatial data consists on describing spatial entities using their qualitative characteristics instead of metrical ones. This description led to represent and reason on spatial data without the need to know their precise shape and position; the research field that deals with qualitative characteristics is known as Qualitative Spatial Reasoning (QSR).

Research in QSR is motivated by a wide variety of applications including Geographic Information Systems (GIS), robotic navigation, high level vision and natural language interpretation [6]. In GIS applications there is the need to abstract from the numerical data and specify a query in a way that is essentially qualitative. Examples of research employing QSR include qualitative reasoning about shape such as [4]. Regarding robot navigation tasks, an approach in the development of a robust qualitative method for robot exploration, mapping and navigation is described in [14].

There are different aspects of space and therefore to its representation; it is important to define what kind of spatial entity to admit for a specific application and also to consider different ways of describing the relationship between these kinds of spatial entities.

It is natural to represent qualitative information using relations. Over the years, several qualitative spatial calculi have been developed to reason with qualitative spatial information. Most of them concentrate on reasoning about one particular aspect of space, for instance topology [8] [18] or position; reasoning about position can be classified to reasoning about configuration of point objects [11] [10] [19], lines [20] [17] or rectangles [13].

Usually a system of qualitative relationships between spatial entities is developed, which covers the particular spatial aspect that is of interests to a degree that appears useful from an applicational or a cognitive point of view.

A description method to represent spatial objects and relations among them makes use of ontologies. In theory, an ontology is a "formal, explicit specification of a shared conceptualisation" [12] and provides a shared vocabulary, which can be used to model a domain that is, the type of objects and/or concepts that exist, and their properties and relations [3]. In other words an ontology is a formal representation of a set of concepts within a domain and the relationships between those concepts [2].

For some purposes it is enough to only have a representation of spatial knowledge, but in more general Artificial Intelligence applications one also needs to reason on given information.

There are different reasoning tasks an intelligent system has to perform: deriving new knowledge from the given one, checking consistency of given information, updating the given knowledge, find a minimal representation.

According to [6], the main research topic in spatial reasoning include determining the complexity of reasoning over different spatial calculi, proving if a formalism is decidable, developing efficient algorithms for spatial reasoning, computing composition tables and verifying their correctness, determining whether a qualitative spatial description is realizable.

Reconstruction involving qualitative information has so far mainly been investigated on a purely qualitative level. An example of reasoning on different qualitative spatial calculi for object configuration reconstruction is presented in [21]. In [16] a qualitative reasoning is described that deals with the problem of ambiguous local landmark observations that have to be integrated into survey knowledge. Finally [24] presents an approach that makes use of qualitative spatial reasoning to check satisfiability of graph-based topological representations for mobile robot tasks. However, there exists little work that involves both qualitative and quantitative information.

4 Contribution

As stated in the application scenario section, the proposed work will take into consideration only some of the described tasks involved in the map reconstruction operation.

In particular it is supposed to have a description of environmental entities that can be both qualitative and quantitative. Some entities are fully described using quantitative characteristics (e.g. dimension, position, etc.) while other entities are described using qualitative characteristics (e.g. relative size, relative position respect to other entities, etc.). Also qualitative relations among entities are available. It is not important, for the target of the proposed work, how this information is retrieved. It is also supposed that quantitative and qualitative descriptions of the environment are stored and managed by a system that allows to retrieving information both qualitative and quantitative in an easy and computationally efficient way.

Two particular situations can be analysed when the data is described only using qualitative or quantitative characteristics. Within the proposed work mainly the general case will be considered; namely the general situation is the one in which data is described using both quantitative and qualitative characteristics.

This integrated approach can be used to accomplish different reasoning operations to retrieve new information from the given ones. In particular two different tasks will be described in the following sections: derivation of new quantitative information and derivation of new qualitative information.

These tasks contribute to solving different requirements regarding two higher level operations: the visualization operation and the querying operation.

The visualization operation regards how given information can be depicted, e.g. in a map, to be useful to final users for example for navigation requirements.

Querying operations can help final users to accomplish different requirements, one example can be given by the decision making process in which the user would like to have, from the system, information to decide if a choice is better than another one, e.g. for path planning tasks.

In Figure 1 a block schema of the proposed system is depicted.

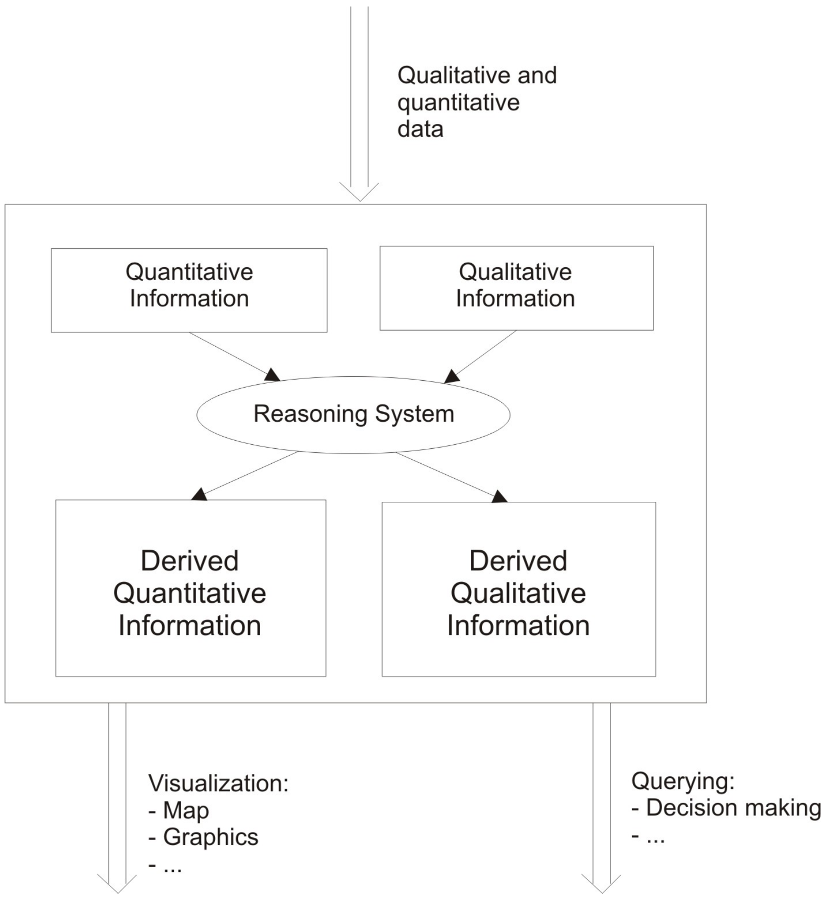

Figure 1: Block Scheme

4.1 Quantitative information derivation

The aim of this task is to derive new quantitative information starting from the integrated qualitative and quantitative description of the environment. This can be helpful to create a quantitative description of unknown entities.

In the following example we will consider two different kinds of qualitative relations among spatial objects.

The first set of relations considered is the one defined in the 5-Intersection model [5], that defines ternary projective relations between regions, starting from the basic concept of collinearity among points. In the case of regions, Clementini and Billen subdivided the plane using the internal and external tangents between the reference objects B and C. Figure 2 depicts the resulting five acceptance areas: BF(B;C) (before), BT(B;C) (between), AF(B;C) (after), LS(B;C) (leftside) and RS(B;C) (rightside).

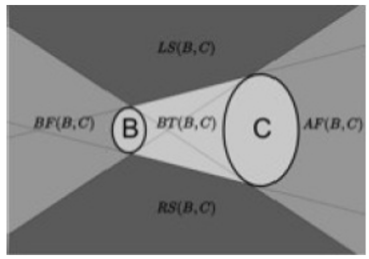

Figure 2: 5-Intersection Model

The second set of relations is the one defined in the Visibility model [22], that defines ternary projective relations between regions. As in the previous model the plane is subdivided into acceptance areas using the internal and external tangents between the reference objects B and C. Figure 3 depicts the resulting three acceptance areas: LZ(B;C) (Light zone, corresponding to the relation Visible(A,B,C)), TZ(B;C) (Twilight zone, corresponding to the relation PartiallyVisible(A,B,C)), and SZ(B;C) (Shadow zone, corresponding to the relation Occluded(A,B,C)).

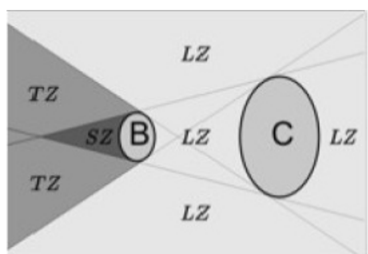

Figure 3: Visibility Model

Take as an example the three regions A, B and C drawn in Figure 4.

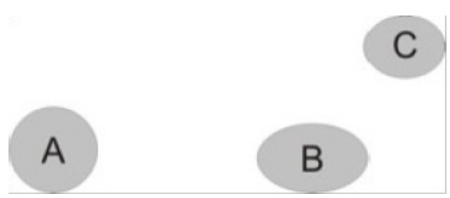

Figure 4: Example Regions

Then, we can suppose to have a fourth region X, which real extension or position are not known, but that in relation with the items described above satisfies the following statements:

- X is between A and B
- X is partially visible from C, considering B as an obstacle

These two expressions, formulated in natural language, could be modelled using the 5 Intersections model [5] and the Visibility one [22]. The formalization of these expressions might be:

- PartiallyVisible(X,B,C)
- Between(X,A,B)

Using the above cited models, the X object must lie in the overlapping zone depicted in Figure 5 to satisfy the given constraints.

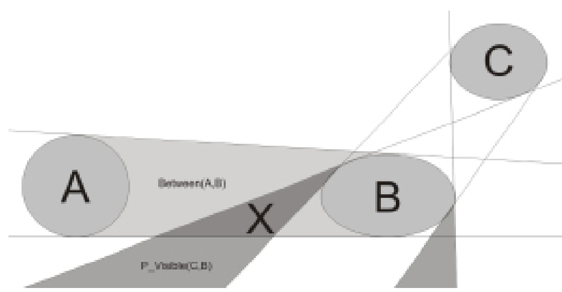

Figure 5: Example Regions

As a first instance we can approximate the object X supposing that it is equal to the area in the figure.

Obviously the given example describes a simple case of quantitative information derivation. Targets of the proposed work are to investigate which kind of qualitative descriptions and relations are useful to describe the environment to accomplish the quantitative reconstruction operation and to solve the problem in the general case in which more than one object is described in qualitative way. The approximated object has to be as similar as possible to the real object.

4.2 Qualitative information derivation

Instead of deriving quantitative information, it is possible to derive new qualitative descriptions and relations among entities to use for further reasoning tasks.

The following example will show how this task could be accomplished in a simple case. The example will make use of topology relations [8] and of relations introduced by the 5 Intersection Model [5]. Let's consider to have a geometric description of the regions A,B, C depicted in Figure 6.

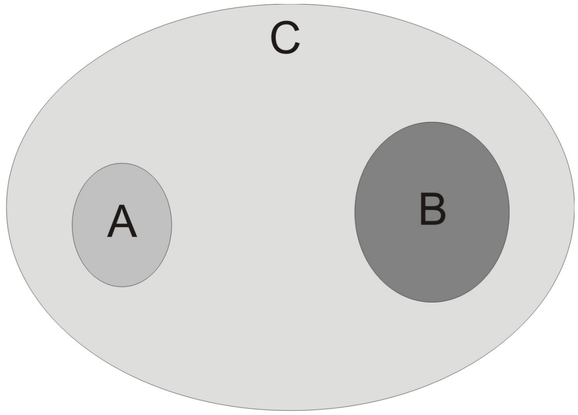

Figure 6: Example

From the given geometries is possible, in an easy way, to infer that regions A and B are contained in the region C.

If a region X is described in a qualitative way by the relation:

- Between(X,A,B)

it is possible to infer that also that the region X is contained in the region C, as depicted in figure 7.

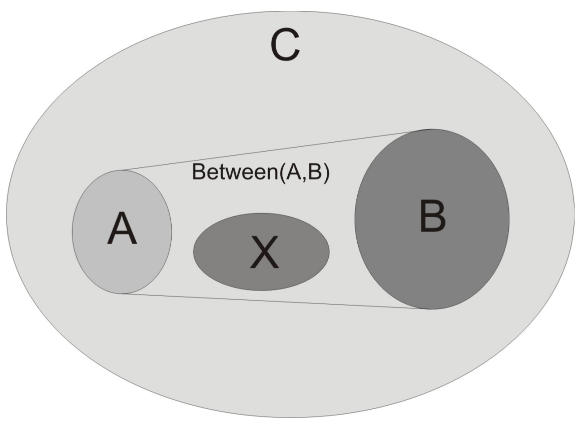

Figure 7: Example

Also this example describes a simple case in which it is easy to infer new qualitative information starting from an integrated qualitative and quantitative description of the environment. The target of the proposed work is to develop reasoning methods to infer new qualitative information in the general case in which entities are in more complex configurations, respect to the one given in the example.

4.3 Further problems involved

Within the proposed tasks some common requirements have to be taken into consideration.

First of all the processes that led to infer new information, both qualitative and quantitative, have to be efficient regarding time performances. This means that it is not sufficient to find methods to infer new information, but these methods have to furnish good solutions for developing applications to solve problems of the real life and not only in a theoretical way.

It has also to be considered that the given information can be inconsistent. This means that some of the given information could be in conflict with other information. In this case, considering all the given information, there should be no solution for example to the task of geometric reconstruction of regions. To let the information inference processes described before to be useful also in this particular case, techniques to catch the inconsistency case have to be found and methodologies to resolve conflicts have to be developed.

Furthermore it is possible to consider the case in which information is underdetermined. In this situation given information is not sufficient to infer new precise knowledge; namely there can be more than one acceptable solution to the considered problem (e.g. in the quantitative reconstruction problem, there can be two different metric representations that respect the given qualitative constraints).

4.4 Contribution summary

In this section the proposed work described above will be briefly summarized.

Target: Infer new spatial information (quantitative and qualitative) starting from a description of the environment that make use of both quantitative and qualitative descriptions. Given information and derived information will be used to solve different tasks like visualization (e.g. maps, graphics, etc.) and querying (e.g. support to decision making problems).

Research contribution: Develop a reasoning system to:

- *Derive quantitative information:* Find out a metric description (e.g. shape and position) for unknown entities starting from the mixed qualitative-quantitative representation of the environment.
- *Derive qualitative information:* Find out qualitative relations among objects starting from the mixed qualitative-quantitative representation of the environment.

Problems involved: Problems to address in the proposed research are:

- *Performances:* the reasoning system has to be efficient regarding time performances.
- *Inconsistency problems:* some of the given information could be in contrast with other ones.

- *Information undetermined:* there could be more than one solution to the given problem.

5 Approach

In this section a practical approach to solve tasks described before is presented.

Obviously the first step regards how spatial data can be represented; this means that it is necessary to know how geometric data is represented (for the quantitative description) and which are the characteristics of the qualitative spatial calculi developed during the years (for the qualitative description).

After that some particular calculi have to be chosen to describe object in the given domain. Particular characteristics of the application scenario have to be taken into consideration.

If some characteristics that seem to be crucial for the description of the environment could not be described using existing calculi, the possibility to develop new spatial calculi to describe the particular characteristic need to be investigated.

Some other information for the description process can be added taking care of some physical characteristics of the objects of the domain (e.g. the maximal width of a road). These characteristics have to be described in a formal way to be useful for the reasoning process.

After the entities description (qualitative and quantitative) task will be accomplished, reasoning methods to derive new information have to be developed.

The first assumption will be to not consider the inconsistency problem described in the previous section. This will simplify, as a first approach, the step of deriving new information.

An hybrid framework for a mixed quantitative-qualitative reasoning will be developed to derive new information. The computational efficiency of algorithms will be taken into consideration. Regarding the quantitative approximation of entities it is also possible to find out techniques to evaluate the approximation quality.

After that it is possible to introduce the consistency problem, finding a way to catch the presence of inconsistencies in the given data. Algorithms founded before have to be adapted to this situation and techniques to make a constraint relaxation have to be studied.

During the whole process described above, I want to develop an exemplary application to implement the reasoning techniques developed in a theoretical way. Such an application will furnish instruments to make practical examples in the given domain, namely the earthquake scenario. It is not in the scope of the proposed work to solve problems about how to store quantitative and qualitative information, so one assumes that the application will lie on an already existing system that deals with this kind of problems. The main target of the application is to implement the reasoning system on the mixed qualitative and quantitative representation, to derive both quantitative and qualitative new information.

The application will be useful also to validate theories that will be developed during the whole proposed work.

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